STUDENTS' ABSTRACTION IN SOLVING SYSTEM OF LINEAR EQUATION IN TWO VARIABLE

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Abstract
The objective of this research is to describe students' abstraction in solving system of linear equation in two variable. Refers to research objective, the data was analyzed based on qualitative method. The data is gained from result of solving contextual problem in worksheet and from interview on students. The research finding shows that students can solve system of linear equation in two variables in formal symbolic by abstraction which is shown in this paper.

Introduction

System of linear equation in two variable (SLETV) had been learned in second year junior high school. The basic competence that students will achieve after learning is to explain the solution of SLETV in contextual problem. Students' experience of resolving contextual issues becomes the path to achieving basic competencies in particularly understanding the concept of solving SLETV in formal symbolic.

In teaching learning process usually the teacher guides students only to follow the demonstrated procedure and its result in the students not having the opportunity to high level thinking, since the curriculum hope teaching practices lead students to high levels thinking that students can demonstrate logical, critical, creative, and innovative thinking skills.

Knowledge is not given to students in the ready made product, but the student must construct it for himself [1]. If students are invited to learn by constructing, students are expected to think critically or creatively. Piaget's note that "*All new knowledge presupposes an abstraction, ...*". [2]. All new knowledge requires abstraction. The statement shows that without abstraction there will be no new knowledge.

To improve long-term conceptual learning, the framework formulated here suggests that the whole curriculum must be framed with an awareness of the abstraction process to produce thinkable concepts at every stage [3]. Understanding the concept must go through the process of abstraction step by step.

In constructing knowledge is an abstraction activity. Abstraction is the process in which the learner construct a new knowledge, by what we call vertical mathematization. Abstraction is the process of constructing new knowledge with vertical mathematization, through recognizing and assembling previously possessed knowledge, and the knowledge identified and assembled must be relevant to the new knowledge to be constructed [4]. Vertical mathematization of terminology in realistic mathematics education is a process of increasing the complexity of a concept in the mathematical system itself. Horizontal mathematization is the process of transforming understanding from real situations to mathematical situations. In this research, The abstractions observed and those that appear in students' mental are three components: 1) recognizing, 2) building-with, and 3) constructing.

In the process of abstraction solving this SLETV will be focused on observation: 1) recognizable element of knowledge that is the initial knowledge possessed by student in solving SLETV, 2) the element of knowledge that is strung together is combination of recognized element, for example solving routine problem (not solving problem which new for the students), and last as a focus is 3) construct the new knowledge for the student which is the answer of the problem. In solving this problem formed a new knowledge or concept expressed in objective learning.

Some researchers stated that abstraction plays an important role in constructing mathematical knowledge and they have shown how one can abstract knowledge empirically, unfortunately they have not shown how students abstract solving SLETV. This research aims to describe students construct the completion of SLETV with abstraction stage. Theoretically this useful research adds to the input on the theory of abstraction can be used in building knowledge in SLETV material. Practically the input on the learning manage as an alternative designing the learning with abstraction theory on the SLETV.

Method

Description of student abstraction in solving SLETV problem required qualitative research. In accordance with the SLETV topic to be solved through abstraction, and based on the applicable curriculum, that the topic of SLETV is taught in grade 8 junior high school, the subjects of this research are a seventh grade student of junior high school and they have not studied formal SLETV material, but have understood the linear equation of one variable. Data collection is done by gives the 3 SLETV problem in each worksheet contained pictures to the students. The 3 SLETV had ben validated to students and shown that problems can be anderstood and can be solved by them. Students give answer, answer written interviewed in depth and validated written and oral answer by seeing consistency of answer. The data collection was assisted audio visual recorder and then the data obtained transcribed and made analysis. The conclusion is made by giving meaning and explanation to the data that has been analyzed.

Result

The following description describes of how the abstraction take places while solving three SLETV problems is contained in the student worksheet. In worksheet 1 the subjects are given a picture containing two packets to be sold at a store. Packet A includes 1 glass and 2 hats for Rp260,000 and packet B contains two glasses and 2 hats for Rp280,000. Subjects are asked to determine the price for each item.

The subjects recognize the solution of the SLETV in their own way after observing the given problem in the picture. Their solution be categorized as the recognition the problem because the subjects complete it based on their experience. This solution has not requested the use of variables yet.

Subsequently subject are asked to write mathematical model by giving the price of a one glass is x and the price of one hat is y. Subjects recognized the mathematical model of the given problem by changing the image or visual form problem into a formal symbol form with the use of x and y variables. Initially the subjects did not add the corresponding variable in the mathematical model, but exposing the process in the figure. For example packet A becomes $x + y + y = x + 2y = 260.000$ and does not directly write $x + 2y = 260.000$. Similarly with packet B the subject writes $x + x + y + y = 2x + 2y = 280.000$. Likewise, the writing of equations is not written in one line, but into three lines. In this

solution, subject had built-with the combination of the use of model and the solution by him self. In this worksheet 1 the subject had not contructed a new knowledge, because there was not obstacle on student working. On the subject's answer, the subject writes x = (2x + 2y) - (x + 2y) = 280,000 - 260,000 = 20,000, for packet A, and the subject still writes 2y = (x + 2y) - x = 260,000 - 20,000. 2y = 240.000 and this subject's text indicates that algebra reduction is already recognized.

The more complex problem prepared in worksheet 2, the subject also is given a picture containing 2 packets to be sold at a store. Packet A contains 1 glass and 2 hats its price Rp220,000 and packet B contains 2 glasses and 3 hats for Rp360,000. Subjects are asked to determine the price for each item.

Problem worksheet 2 can not be solved by solving problem in worksheet 1, because the structures is different. Subject does so by trial and error with the price of 1 hat and the price of a 1 glass, then testing the truth of the price given by substituting the figure on the problem. The subject get the price of 1 glass is Rp60,000 and a 1 hat price of Rp80,000. Another subject has the answer by forming a "new packet" that is the difference between packet B and packet A; the price of 1 hat and one glass Rp140.000. This new packet combines with packet A to get the price of a hat. Packet A price 2 hats and 1 glass Rp 220.000 along with "new packet" price 1 hat and 1 glass Rp140.000. So the subjects get the answer of the price of 1 hat as the difference of the two packets.

In changing the SLETV in the form of the picture and its description into the form of a formal symbol, the subject can recognize the mathematical model by writing x + y + y = x + 2y = 60,000 + 160,000 = 220,000 of packets A and 2x + 3y = 120,000 + 240,000 = 360,000.

To find out the subject of how to building-with to solve this problem 2, it is obtained through the subjects answer in section 2c "Write down your solution by using x and y". Building-with in this section is to combine the solution obtained by the subject through problem 2a and 2b that is the combination of the solution by the subject itself and the mathematical model of the given SLETV. Solution 1b with 2b is about writing the mathematical model, to be different. Subject does not write the price per unit while solving problem 1b, since on problem 2b, the subject writes the prices x, 2y, 2x, and 3y. These prices are obtained through the answer on problem 2a, and the subject has written SLETV in formal symbol.

The subjects exposed his work in front of the class by telling "*Now we see packet B is 360,000 and packet A is 220,000. So packet B minus packet A is 140,000. One hundred and forty its still the price of 1 glass and 1 hat. It is still the price of 1 glass and 1 hat. We want to find the price of each hat and glass. While in packet A there is 1 glass and 2 hats that cost 220,000. One hundred forty thousand, sorry. So packet A is worth 220,000. It is subtracted by 140,000 is equal to 80. The price of 1 hat and 1 glass is 140,000 and the price of 2 hats and 1 glass is 220,000. Two hats and one glass less than one hat and one glass with one hat. One hat is80,000"*. The subject then determines the price of a glass with the following exposure. "*One hat is 80.000. So if the two hats are 160,000. Two glasses and 1 glass is 220,000. So 1 glass is 220,000 minus 160,000 equals to 60,000* ".

Exposure to this subject is already the construct a new knowledge for him even though he did not finish it in a formal symbol. Particularly his ability contruck a new statement from two packets A and B, by taking the difference. The completion of this subject has also used elimination, ie eliminating the price of a glass to obtain the price of a hat. This subject will not determine the solution directly from the SLETV form of formal

symbols with the reason that he has gotten it. In that solution the subjects perceive variables on SLETV have meaning as contents of the original problem. His understanding of SLETV does not show meaningless variables.

The problem in worksheet 3 is more complex than in worksheet 2. Packet A contains one glass and two hat price is Rp300,000 and Packet B contains four glasses and one hat price is Rp360,000. Problem 2 can be solved by 2 stages, first by subtracting one packet from another, then eliminating one variable. packet such as packet A is subtracted with packet B. In worsheet 3 it could not be solved like problem in worksheet 2.

The subject recognizes this SLETV problem but can not recognize the solution yet. The subject recognizes the purpose of the problem in formal symbol. A sample of the subject written "*A* packet: x + y + y = x + 2y = 300.000 and *B* packet: x + x + x + y = 3x + y = 525.000". The subject identified elements to be built-with is in this SLETV solution process even though the subject has not found the solution. For example a combination of problems in the form of picture and their descriptions and combined with a formal symbol. Also it found elements are built-with between integer operations and meaning in the picture.

The subject experienced an obstacle while solving the SLETV in problem 3. This obstacle is shown by their inability to show the answer. Subjects who try to determine the price of a one glass and the price of a hat did not find the answer. It shows that this problem is more complex than the previous problem. Many subjects try to subtract packet A from packet B or vice versa, but fail in interpretaation.

To overcome the obstacle of subject to solve the problem, the interviewer scaffolded subject by giving question as follows. "*Can it be solved if there are some similar packets?"*. The Subject could describe his solution by making 3 similar A packet, solved it like problem in answersheet 2, and found the price of glass is 150,000 that is x = 150 and the the price of hat is 75,000 that is y = 75.

The subject can solve the problem after the researcher made a scaffolding. The interviewer did not provide direct help, but with that question, the subject became triggered to construct his knowledge by completing a new SLETV for him.

Discussion

The problem given to the subjects are a problem they can image or a problem in contextual form, so they attempt to solve the problem packet in SLETV. It is possible if the problem is not close to the environment, or directly SLETV problem in the form of a formal symbol, then the subject will not be motivated to solve it, because subjects do not see the problem for what benefits. Through a given contextual problem, subjects have informal knowledge directed toward formal knowledge. This corresponds to one of the contents of the theory of realistic mathematics education that the child's learning begins with a real situation for the student.

At the solution of the subject using the x and y symbols, in the mind of the subject that x is the price of a single glass, and y is the price of a hat. The variable x and y is a variable meaning, ie a value, and it is not just a letter that has meaningless, although the subject often calls a glass while pointing or writing the letter x and one skilled hat pointing or writing the letter y. Understanding of the subjects for this letter accord with the first stage of understanding of variables according to Kucheman [5] that variables is a letter that will be determined its value.

There is a new knowledge is constructed by the subject while solving SLETV in worksheet 2. Usually the SLETV solution is generally doubled packet A so that it gets 2 glasses and 4 caps, then calculates the difference of packet A duplicated with packet B to obtain the price of one cap. Activity of the subject as demanded by the Curriculum 2013 is student-centered learning so that students find themselves new concepts. This finding is in line with the opinion of Glasersfeld [2] that the knowledge given is not in the finished form, but the student must construct that knowledge for himself. Activity of this subject is an abstraction of knowledge possessed formally and informally, so that constructed a new knowledge for student self. This is in accordance with Piaget [2] that all knowledge is based on abstraction.

In solving the problem SLETV number 3, subjects' experience obstacles, the procedures they use on problem number 1 or number 2 can not be used to solve the problem in number 3. Ozmantar and Monaghan [5] said that student is not easy to contruct new knowledge, the teacher needs do scaffolding. In order to subjects to construct this new knowledge according to them, the researcher provides minimal assistance, by asking "Can it be solved if there are 3 similar A packets?". This question is a trigger for the subject to expose his potential to solve SLETV problems. In accordance with Vigotsky's theory that one has actual abilities and potential abilities and the area between these two abilities is expressed by Zone of Proximal Development [6]. In order to the subject to arrive at this potential ability, a more mature person such as a peer or teacher as a more mature person provides scaffolding.

Conclusion

This research aims to explain how students perform abstraction in solving SLETV. The SLETV problem is presented in the form of a story or contextual problem, so students can recognize and even may have problems so challenged to solve the problem. Through a given contextual problem, subjects have informal knowledge directed toward formal knowledge. In completing the SLETV, subjects recognize the knowledge they have through their own settlement and write symbols informally, then directed by formal symbols, so that students can find the formal symbol (mathematical model) of the SLETV problem. Students need support in constructing new knowledge through solving SLETV problems. For further research, observations of students performing abstractions need to be done on more complex SLETV topics to form formal symbols. In the learning SLETV in the classroom to student-centered with the method of discovery with minimal guidance so that learning is more meaningful.

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